

Dynamics of confined gluons

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Abstract

Propagation of gluons in the confining vacuum is studied in the framework of the background perturbation theory, where nonperturbative background contains confining correlators.

Two settings of the problem are considered. In the first the confined gluon is evolving in time together with static quark and anti-quark forming the one-gluon static hybrid. The hybrid spectrum is calculated in terms of string tension and is in agreement with earlier analytic and lattice calculations. In the second setting the confined gluon is exchanged between quarks and the gluon Green's function is calculated, giving rise to the Coulomb potential modified at large distances. The resulting screening radius of 0.5 fm presents a serious problem when confronting with lattice and experimental data. A possible solution of this discrepancy is discussed.

1 Introduction

Gluons are known to be confined but this property is never taken into account in the Standard Perturbation Theory (SPT). As an argument one refers to the small distance (high momentum) domain where SPT is assumed to be valid. Beyond this domain the SPT displays unphysical singularities, and moreover the very notion of gluon should be properly defined. This can be done in the framework of the Background Perturbation theory (BPT) [1]. The formalism of this kind where the background is assumed to be nonperturbative with confining properties, was developed in [2, 3].

One immediate consequence of this new BPT is that all gluons are confined and moreover the unphysical singularities of SPT (Landau ghost poles and branch points as well as IR renormalons) are removed from the theory [2, 3].

The confined gluons can form several types of systems: glueballs [4], hybrids [5, 6] and gluelumps [7]. The analytic calculations in the quoted papers predict spectrum which in all cases is in good agreement with lattice data, and has a very simple form depending only on string tension σ and α_s .

The main subject of the present paper is the study of the gluon exchange interaction between Q and \bar{Q} when gluon is confined, which can be called the confined Coulomb interaction $V_c^*(R)$. One can expect that at small distances, say $R < 1 \text{ GeV}^{-1}$, the confined Coulomb potential coincides with the standard Coulomb potential $V_c(R) = -C_2 \frac{\alpha_s(R)}{R}$, $C_2 = \frac{N_c^2 - 1}{2N_c}$.

At large R the confined gluon is expected to produce the screening of the Coulomb interaction. At the first glance the screening mass should coincide with the lowest hybrid mass of the static hybrid. However as will be shown below, this naive expectation fails, since gluon propagation between quarks goes not only in time (where hybrid mass is in proper place) but also in distance R (where in addition asymptotics of wave functions enters). As a result one obtains a more complicated behaviour which we display both numerically and analytically.

The plan of the paper is as follows. In section 2 the general formulas of BPT are written down and approximations are discussed. In section 3 a simple toy model is suggested to illustrate method and possible qualitative outcome.

In section 4 the method is applied to calculate the static hybrid Green function and spectrum in a way different from used before, in [6]. In section 5 results of previous section are used to calculate the Green function of the confined gluon exchanged between static quarks, and the resulting screened Coulomb potential. Physical consequences and prospectives are discussed in the concluding section.

2 Dynamics of a confined gluon. Generalities.

In the Field Correlator Method (FCM) the dynamical picture of a confined gluon is simple and selfconsistent: the gluon (its corresponding field is a_μ) moves in the strong and disordered vacuum field B_μ characterized by the correlators of the field $F_{\mu\nu}(B)$, so that the total gluonic field A_μ can be written as

$$A_\mu = B_\mu + a_\mu. \quad (1)$$

Here the problem of separation of A_μ into B_μ and a_μ , and that of double counting is resolved technically by the use of the so-called 'tHooft identity [3] and one can integrate and average over both DB_μ and Da_μ , so that the partition function is

$$Z = \frac{1}{N'} \int DB_\mu Z(J, B), \quad Z(J, B) = \int Da_\mu \exp(-S(a + B) + J(a + B)) \quad (2)$$

and S is the standard QCD Euclidean action including ghost and gauge-fixing terms (see [2, 3] for details).

In what follows we shall be interested in the gluon propagation in the field of static (confined) quark Q and antiquark \bar{Q} . The starting point for the gauge-invariant study of this process is the total Green's function of $Q\bar{Q}$ system, which is proportional to the Wilson loop:

$$\langle W(A) \rangle \equiv \langle \langle W(B + a) \rangle \rangle_{B,a} = \langle \langle W(B + a) \rangle_a \rangle_B, \quad (3)$$

$$W(A) = \text{tr } P \exp ig \int_C A_\mu dz_\mu. \quad (4)$$

The Wilson loop is assumed to be the closed rectangular contour $R \times T_0$ in the (x_1, x_4) plane.

One may expand in ga_μ keeping B_μ intact and this will give the Background Perturbation Theory (BPT) series ([1, 2], see [3] for details), the first terms being

$$\langle W(B + a) \rangle_{B,a} = \langle W^{(0)}(B) \rangle_B - g^2 \langle W^{(2)}(B, x, y) G_{\mu\nu}(x, y) \rangle_B dx_\mu dy_\nu + \dots \quad (5)$$

For the chosen contour C and the Feynman gauge of field B_μ and FFSR for $G_{\mu\nu}$, $W^{(2)}$ and $G_{\mu\nu}$ can be written as

$$W^{(2)} = C_2(f) \Phi_{\alpha\beta}^{(-)}(x, y) \Phi_{\alpha'\beta'}^{(+)}(y, x) \quad (6)$$

$$G_{\mu\nu}(x, y) = \int_0^\infty ds (Dz)_{xy} e^{-K} \left\{ \Phi^{(adj)}(x, y) P \exp(2g \int_0^s F_{\rho\sigma}(z(\tau)) dz) \right\}_{\beta\alpha, \beta'\alpha'}^{(\mu\nu)} \quad (7)$$

where $\Phi^{(\pm)}(x, y)$ are the future/past pieces of the Wilson loop $W^{(0)}(B) = P \exp(ig \oint B_\mu dz_\mu)$, obtained by cutting it at points x and y . Note that the adjoint phase factor $\Phi^{(adj)}(x, y) \equiv \exp(ig \int_y^x B_\mu dz_\mu)$ from $G_{\mu\nu}$ which can be written as the product

$$\Phi^{(adj)}(x, y) = \Phi_{\beta\alpha}(y, x) \Phi_{\beta'\alpha'}(x, y) \quad (8)$$

the total construction produces two closed Wilson loops (see ref. [2, 3, 6] for pictures and discussion).

$$\Phi^{(-)} \Phi^{(adj)} \Phi^{(+)} = W_\sigma^{(-)}(x, y) W_\sigma^{(+)}(y, x). \quad (9)$$

Note the subscript σ in (9) which implies the color-magnetic spin factor (the last factor on the r.h.s. of (7)) entering in all Wilson lines including $\Phi^{(adj)}$. The averaging of (9) over fields B_μ can be easily done at large N_c

$$\langle W_{\mu\nu}^{(2)} G_{\mu\nu} \rangle_B = G^{(0)} \langle W_\sigma^{(-)}(x, y) \rangle_B \langle W_\sigma^{(+)}(y, x) \rangle_B, \quad (10)$$

where $G^{(0)}$ denotes the integral $\int ds (DZ)_{xy} e^{-K}$. In the FCM one obtains for $\langle W \rangle$ the area law behaviour at large distances $R, T \gg \lambda$

$$\langle W \rangle = \exp(-\sigma S_{min}) \quad (11)$$

and λ is the gluon correlation length [8], characterizing the fall-off in x of the field correlator $D(x) \sim \langle \text{tr} F(x) \Phi F(0) \rangle$, and S_{min} is the area, which is assumed to be minimal for the given contour C .

Note that at large distances the spin factor does not contribute to the area law [4-7] and therefore the subscript σ in (11) is omitted.

To describe S_{min} one can parametrize first the trajectory $z_\mu(t)$ of the gluon between the initial point x and the final point y (both on the contour C).

$$z_\mu : (z_1(t) \equiv \xi(t), \quad z_2(t), \quad z_3(t), \quad z_4(t) \equiv t), \quad h^2(t) = z_2^2(t) + z_3^2(t) \quad (12)$$

We choose the Nambu-Goto ansatz for the minimal area surface (or rather for its increase over the plane area $S_0 = RT$)

$$\Delta S = \Delta S_1 + \Delta S_2, \quad \Delta S_i = \int_{y_4}^{x_4} dt \int_0^1 d\beta \sqrt{(\dot{w}_i w'_i)^2 - \dot{w}_i^2 w_i'^2} \quad (13)$$

where $w_{i\mu}(\tau, \beta) : w_{i4} = t, \mathbf{w}_{1,2} = (1 - \beta)(\mathbf{R}, 0) + \beta\mathbf{z}$. Note, that in our case two different processes can be initiated by the exchanged gluon:

i) points x and y are at $x_4 = T_0, \mathbf{x} = (\frac{R}{2}, 0, 0)$ and $y_4 = 0, \mathbf{y} = (\frac{R}{2}, 0, 0)$, the situation which is insured on the lattice by the insertion of plaquettes at these sides of Wilson loop. In this case one obtains the hybrid excitation of the Wilson loop, and this form was used before to compute hybrid spectra analytically [5, 6] and on the lattice [9].

ii) the points x and y belong to the trajectories of Q and \bar{Q} respectively, so that $G_{\mu\nu}(x, y) = G_{44}(R, T), T < T_0$, describes the propagation of Coulomb gluon between the quarks.

In what follows we shall study both cases using for that the final form resulting from (5), (7), (11), (13)

$$\langle G_{44}(x, y) \rangle_B \equiv G(x, y) = \int_0^\infty ds (Dz)_{xy} e^{-K - \sigma \Delta S} \quad (14)$$

To treat the awkward roots in ΔS , Eq. (13), one can introduce the einbein parameters $\nu(t)$ and $\bar{\nu}(t)$ [10] obtaining in the small oscillation limit

$$G(x, y) = \int_0^\infty ds (Dz)_{xy} D\nu D\bar{\nu} e^{-K - \sigma \Delta \tilde{S}(\nu, \bar{\nu})} \quad (15)$$

where

$$\Delta \tilde{S}(\nu, \bar{\nu}) = \int_{y_4}^{x_4} dt \left\{ \frac{\nu + \bar{\nu}}{2} \left(1 - \frac{1}{3} \dot{z}_\perp^2 \right) + \frac{h^2 + \xi^2}{2\nu} + \frac{h^2 + (R - \xi)^2}{2\bar{\nu}} - R \right\} \quad (16)$$

For $x = (\frac{R}{2}, 0, 0, T_0); y = (\frac{R}{2}, 0, 0, 0)$ one will have from (14) the static hybrid spectrum, to be compared with previous calculations, and for the case ii) one can define the generalized Coulomb interaction as

$$\int_0^{T_0} dx_4 \int_0^{T_0} dy_4 G(x, y) = \int_0^{T_0} d\frac{x_4 + y_4}{2} \int_{-T_0/2}^{T_0/2} G(R, T) dT \equiv T_0 V_C^*(R). \quad (17)$$

In the limit $\sigma \rightarrow 0$ one obtains the free gluon propagator $G^{(0)}(x, y) = \frac{1}{4\pi^2(x-y)^2}$ in case i) and the Coulomb interaction $V_c^*(R) = V_c(R) = -C_2 \frac{\alpha_s}{R}$ in the case ii). Our purpose in what follows is to obtain modification of these results for nonzero σ . To grasp the idea we shall start with a toy model as a warm-up.

3 A simple toy model for the confined gluon

Consider a gluon propagating from static quark Q with coordinate $(0, \vec{0})$ to antiquark $\bar{Q}(T, R, 0, 0)$.

The world sheet of $Q\bar{Q}$ system with the string from Q to \bar{Q} sweeps the strip in the (x_4, x_1) plane, and one expects that gluons are confined dynamically to some region around this strip. This means that the running away of gluons from the plane (x_4, x_1) is damped by some function, which we take in the form of the confining "potential" V ,

$$V(\mathbf{z}) = \frac{\omega^2}{4}(z_2^2 + z_3^2). \quad (18)$$

The Green's function of the gluon can be written in the FFSR [11]

$$G(T, R) = \int_0^\infty ds (Dz_1)_{0R} (Dz_4)_{0T} (Dz_2)_{00} (Dz_3)_{00} e^{-K - \int_0^s V d\tau} \quad (19)$$

where $K = \frac{1}{4} \int_0^s d\tau \left(\frac{dz_\mu}{d\tau} \right)^2$, and $(Dz_i)_{ab} = \prod_{n=1}^N \frac{dp_i}{2\pi} e^{ip_i(a_i - b_i)} \left(\frac{d\Delta z_i(n)}{4\pi\varepsilon} \right)$.

The integration in Dz_i factorizes and can be performed immediately, with the result (see Appendix 1 for details)

$$G(T, R) = \frac{1}{16\pi^2} \int_0^\infty \frac{ds}{s^2} \varphi(\omega s) \exp\left(-\frac{x^2}{4s}\right), \quad \varphi(t) = \frac{t}{\sinh t}; \quad x^2 = R^2 + T^2. \quad (20)$$

The integral (20) can be estimated by the stationary point method and one obtains

$$G(T, R) \cong \frac{\psi(\omega x^2)}{4\pi^2 x^2}, \quad \psi(t) \approx \begin{cases} \frac{t}{8 \sinh\left(\frac{t}{8}\right)}, & t \ll 1 \\ \sqrt{t} e^{-\sqrt{t}}, & t \gg 1 \end{cases}. \quad (21)$$

One can see that at large x^2 , $x^2\omega \gg 1$, there appears in (21) the damping factor signaling the mass gap $m = \sqrt{\omega}$, i.e. the confined gluon acts at large distances as a massive particle, while at small distances, $x^2 \rightarrow 0$, it behaves as the ordinary unconfined gluon. When $\omega \rightarrow 0$ one recovers from (17) the standard Coulomb interaction:

$$V_c(R) = -g^2 C_2(f) 2 \int_0^\infty G(T, R) dT = -\frac{\alpha_s C_2(f)}{R},$$

and in the opposite limit, $\omega R^2 \gg 1$, $V_c(R)$ is multiplied by the factor $\frac{4\sqrt{2}}{(\omega R^2)^{1/4}} \exp(-(\omega R^2)^{1/2})$. This screening factor is equal 1/2 at $R \approx \sqrt{\frac{4}{\omega}}$ and

decays exponentially at large R , as it is commonly expected. In section 5 a more complicated behaviour will be obtained in the realistic case when gluon is confined by the string world-sheet. To this end we develop in the next section the necessary formalism for the hybrid Green function.

4 Spectrum of the confined gluons. Static hybrid

In this section we shall calculate the gluon Green's function in the hybrid situation, i.e. when boundary conditions are given in i) below Eq. (13), and one can identify $T \equiv T_0$. One can introduce the einbein variable $\mu(t)$ as in [10] (see Appendix 2 for details) and write

$$G(x, y) = \int \frac{D\mu}{2\bar{\mu}} D\nu D\bar{\nu} e^{-\Gamma} G_3(R, T, \nu, \bar{\nu}, \mu) \quad (22)$$

where we have defined

$$\Gamma = \int_0^T \frac{\mu}{2} dt + \frac{\sigma}{2} \int_0^T (\nu + \bar{\nu}) dt + \sigma R^2 \int_0^T \frac{dt}{2(\nu + \bar{\nu})}, \quad (23)$$

$$G_3 = \int (D^3 z)_{xy} e^{-\int_0^T \left(\frac{\mu}{2} \dot{z}_1^2 + \frac{\mu + J_1 + J_2}{2} \dot{z}_\perp^2 \right) dt - \frac{\sigma}{2\bar{\nu}} \int_0^T [(z_1 - \tilde{R})^2 + h^2] dt}, \quad (24)$$

$\tilde{R} = R \frac{\nu}{\nu + \bar{\nu}}$, and we take $x = \left(\frac{R}{2}, 0, 0, 0 \right)$, $y = \left(\frac{R}{2}, 0, 0, T \right)$, $J_i = \frac{\sigma}{3} \nu_i$.

The integration over $(D^3 z)$ can be immediately done using the standard path integral formula, see Appendix 1.

$$G_3 = \left(\frac{\mu \omega_1}{2\pi \sinh \omega_1 T} \right)^{1/2} \frac{(\mu + J_1 + J_2) \omega_\perp}{2\pi \sinh(\omega_\perp T)} \exp \left\{ -\frac{\mu \omega_1}{2 \sinh \omega_1 T} \frac{R^2 (\nu - \bar{\nu})^2}{2(\nu + \bar{\nu})^2} (\cosh(\omega_1 T) - 1) \right\} \quad (25)$$

where $\omega_1 = \sqrt{\frac{\sigma}{\mu \bar{\nu}}}$, $\omega_\perp = \sqrt{\frac{\sigma}{\bar{\nu}(\mu + J_1 + J_2)}}$, $\tilde{\nu} = \frac{\nu \bar{\nu}}{\nu + \bar{\nu}}$.

The next step is the stationary point analysis of Eqs. (22), (25) with respect to variables $\nu, \bar{\nu}, \mu$. We relegate the details of this analysis to the Appendix 2 and here only quote the result. Minimizing the expression in the

exponent at (25) with respect to $(\nu - \bar{\nu})$ one obtains that $\nu = \bar{\nu}$, and the stationary point in ν , $\nu = \nu_0$ is found from the equation

$$\frac{\partial}{\partial \nu}(\sigma \nu + \frac{\sigma R^2}{4\nu} + \frac{\omega_{\perp}}{2} + \omega_{\perp}) = 0. \quad (26)$$

One can distinguish two cases: a) $\sigma R^2 \gg 1$; b) $\sigma R^2 \ll 1$.

a) In the case a) taking into account that $\mu_0 \lesssim \sqrt{\sigma}$ (which will be confirmed afterwards), one has

$$\nu_0 = \frac{R}{2}, \quad \omega_1^{(0)} \rightarrow \left(\frac{4\sigma}{\mu R}\right)^{1/2}, \quad \omega_{\perp}^{(0)} \rightarrow \frac{\sqrt{12}}{R} \quad (27)$$

and Eq.(22) with $\nu = \bar{\nu} = \nu_0$ assumes the form for large ωT

$$G(x, y) = \int D\mu \exp[-(\sigma R + \omega_{\perp}^{(0)} + \frac{\mu}{2} + \frac{1}{2}\sqrt{\frac{4\sigma}{\mu R}})T] \quad (28)$$

The stationary point $\mu = \mu_0$ is found from the exponential of (28) to be

$$\mu_0 = \left(\frac{\sigma}{R}\right)^{1/3},$$

and the resulting static hybrid mass at large R is

$$M_{hybrid}(R) = \sigma R + \frac{3}{2} \left(\frac{\sigma}{R}\right)^{1/3} + \frac{\sqrt{12}}{R} \quad (29)$$

The second term on the r.h.s. of (29) is the characteristic " $R^{-1/3}$ law" for large R hybrid excitations, studied in [6], and one can distinguish the longitudinal and transverse branches of spectrum with higher excitations generated by $\sinh \omega_1 T$ and $\sinh \omega_{\perp} T$ in (25), which we write as in [6]

$$M_{hybrid}^{(long)} = \frac{3}{2^{1/3}} \left(\frac{\sigma}{R}\right)^{1/3} \left(n_z + \frac{1}{2}\right)^{2/3}, \quad M_{hybrid}^{(trans)} = \frac{\sqrt{12}}{R} (n_{\perp} + \Lambda + 1), \quad (30)$$

where Λ is angular momentum projection on the x axis. Note that $\sqrt{12} = 3.46 \approx \pi$ and transverse spectrum is very close to the flux tube excitations, while the longitudinal found in [6] is new.

The resulting spectrum (30) is in a good agreement with lattice calculations [9], see also discussion in [6].

b) We now turn to the case b) $\sigma R^2 \ll 1$, and from (26) find that $\nu = \nu_0 = \left(\frac{9}{\sigma\mu}\right)^{1/3}$ and the equivalent of Eq. (28) is

$$G(x, y) = \int D\mu \exp \left\{ - \left[\frac{\mu}{2} + \frac{3}{2} \frac{(3\sigma)^{2/3}}{\mu^{1/3}} + \frac{\sigma R^2}{2} \left(\frac{\sigma\mu}{9} \right)^{1/3} \right] T \right\}. \quad (31)$$

To check accuracy of our approach we can calculate the hybrid mass in the limit $R \rightarrow 0$, which coincides with the gluelump case [7]. Defining for $R \rightarrow 0$ the stationary point $\mu = \mu_0$ from the exponent of (31) one has $\mu = \mu_0 = \sqrt{3\sigma}$ and the gluelump mass is

$$M_{gluelump} = 2\sqrt{3\sigma} \quad (32)$$

which should be compared with the dedicated gluelump calculations in [7]: $M_0 = 2 \left(\frac{a}{3}\right)^{3/4} (2\sigma_{adj})^{1/2} = 2\sqrt{3.096\sigma}$, where we have used the value of the first zero of the Airy function, $a = 2.338$. One can see agreement on the level of 1.5%.

Taking into account the last term in the exponent of (31) one obtains the lowest hybrid mass at small $\sigma R^2 \ll 1$

$$M_{hybrid}(R) = 2\sqrt{3\sigma} + \frac{\sigma R^2}{2} \sqrt{\frac{\sigma}{3}} + O(R^4) \quad (33)$$

which coincides with the mass spectrum obtained in [6] for this limiting case by a different method.

Thus our approach can be used as a good zeroth order approximation for the confined gluon Green's function and its spectrum.

In what follows we shall use the dependence $\mu_0(R) = \bar{\mu}$ given above in two limiting cases:

$$\begin{aligned} \mu_0(R) &= \left(\frac{\sigma}{R}\right)^{1/3}, \quad \sigma R^2 \gg 1 \\ \mu_0(R) &= \sqrt{3\sigma}, \quad \sigma R^2 \ll 1 \end{aligned} \quad (34)$$

5 The confined Coulomb interaction

In this section we consider the confined gluon Green's function for the initial and final conditions corresponding to the Coulomb gluon exchange.

With the same notations as in (22)-(24), one has

$$G(R, T) = \int D\nu D\bar{\nu} \frac{D\mu}{2\bar{\mu}} e^{-\Gamma} G_3^{(C)}(R, T, \nu, \bar{\nu}, \mu) \quad (35)$$

where Γ is the same as in (23), but now $G_3^{(C)}$ is not given by (25), but has another form, due to different initial and final condition: $x = (0, 0, 0, 0)$, $y = (R, 0, 0, T)$ and the same simple general formula of Appendix 1, yields

$$G_3^{(C)} = \left(\frac{\bar{\mu}\omega_1}{2\pi \sinh \omega_1 T} \right)^{1/2} \frac{(\bar{\mu} + J_1 + J_2)\omega_\perp}{2\pi \sinh(\omega_\perp T)} \exp \left\{ -\frac{\bar{\mu}\omega_1}{2 \sinh(\omega_1 T)} [R^2 \cosh \omega_1 T + 2R^2 \frac{\tilde{\nu}}{\nu + \bar{\nu}} (1 - \cosh \omega_1 T)] \right\} \quad (36)$$

where we have defined ω_\perp as in previous section, while

$$\omega_1 = \sqrt{\frac{4\sigma}{\bar{\mu}(\nu + \bar{\nu})}}, \quad D\mu = \prod_n \frac{d\mu(n)\sqrt{\Delta t}}{\sqrt{2\pi\mu(n)}},$$

so that $\int D\mu \exp \left\{ -\frac{1}{2} \int_0^T \mu(t) dt \right\} = 1$. Minimizing in $(\nu - \bar{\nu})$, one obtains $\nu = \bar{\nu}$, $\omega_1 \rightarrow \omega_1^{(0)} = \sqrt{\frac{2\sigma}{\bar{\mu}\nu}}$, $\omega_\perp \rightarrow \omega_\perp^{(0)} = \sqrt{\frac{2\sigma}{(\bar{\mu} + J_1 + J_2)\nu}}$ and one has

$$G(R, T) = \int \frac{D\nu}{(2\pi)^{3/2} 2\bar{\mu}} \exp(-\Gamma_0) \quad (37)$$

where

$$\Gamma_0 = \sigma\nu T + \frac{\sigma R^2 T}{4\nu} + \frac{1}{2} \ln \sinh(\omega_1^{(0)} T) - \frac{1}{2} \ln(\bar{\mu}\omega_1^{(0)}) + \frac{\bar{\mu}R^2}{2T} \varphi(\omega_1^{(0)} T) + \ln \sinh(\omega_\perp^{(0)} T) - \ln((\bar{\mu} + J_1 + J_2)\omega_\perp^{(0)}), \quad (38)$$

and

$$\varphi(x) = \frac{x(1 + \cosh x)}{2 \sinh x}, \quad \varphi(0) = 1, \quad \varphi(x \rightarrow \infty) \approx \frac{x}{2} \quad (39)$$

To proceed one should find the stationary point of Γ_0 with respect to ν , $\frac{\partial \Gamma_0}{\partial \nu} \Big|_{\nu=\nu_0} = 0$. This is easy to do at large R , since then $\nu_0 \sim \frac{R}{2}$ and $\omega_1^{(0)} = \sqrt{\frac{2\sigma}{\bar{\mu}\nu_0}} \rightarrow 0$, $\omega_\perp^{(0)} \rightarrow 0$ so that the last three terms on the r.h.s. of (38) do not contribute.

$$\frac{\partial \Gamma_0}{\partial \nu} = 0 = \sigma T \left(1 - \frac{R^2}{4\nu_0^2} \right), \quad \nu_0 = \frac{R}{2}, \quad R \rightarrow \infty \quad (40)$$

and

$$\Gamma_0(\nu = \nu_0) - \sigma RT = \frac{1}{2} \ln \sinh \left(\sqrt{\frac{4\sigma}{\bar{\mu}R}} T \right) - \frac{1}{2} \ln \left(\bar{\mu} \sqrt{\frac{4\sigma}{\bar{\mu}R}} \right) + \frac{\bar{\mu}R^2}{2T} \varphi \left(\sqrt{\frac{4\sigma}{\bar{\mu}R}} T \right) + \ln \sinh \left(\frac{\sqrt{12}}{R} T \right) - \ln \left(\bar{\mu} + \frac{\sigma R}{3} \right) \frac{\sqrt{12}}{R}. \quad (41)$$

To get the modified Coulomb interaction at large R , one considers the integral

$$V^*(R) \equiv \int_{-\infty}^{\infty} dTG(R, T) = 2 \int_0^{\infty} dTG(R, T) = \frac{2}{(2\pi)^{3/2} 2\bar{\mu}} \int_0^{\infty} e^{-\Gamma_0} dT. \quad (42)$$

Inserting (41) into (42) one obtains

$$V^*(R) = \frac{2\sqrt{3} \left(\frac{\bar{\mu}}{R} + \frac{\sigma}{3} \right)}{2\pi^{3/2}} \int_0^{\infty} \frac{dT}{\sinh \left(\frac{\sqrt{12}T}{R} \right) \left[\sinh \left(\sqrt{\frac{4\sigma}{\bar{\mu}R}} T \right) \right]^{1/2}} \exp \left(-\frac{\bar{\mu}R^2}{2T} \varphi \left(\sqrt{\frac{4\sigma}{\bar{\mu}R}} T \right) \right). \quad (43)$$

Introducing new variable $\tau = \frac{\bar{\mu}R^2}{2T}$ this integral can be reduced to the form

$$V^*(R) = \frac{\sqrt{3} \left((\bar{\mu}R)^2 + \frac{\lambda^2}{3} \right)}{2\pi^{3/2} R} \int_0^{\infty} \frac{d\tau}{\tau^2} \frac{e^{-\tau \varphi \left(\frac{\lambda}{\tau} \right)}}{\left(\sinh \frac{\lambda}{\tau} \right)^{1/2} \sinh \left(\frac{\sqrt{3}\bar{\mu}R}{\tau} \right)} \equiv \frac{\xi(R)}{4\pi R} \quad (44)$$

where we have defined $\lambda = (\sigma \bar{\mu} R^3)^{1/2} \rightarrow_{R \rightarrow \infty} (\sigma R^2)^{2/3}$. Finally $\xi(R)$ can be written as

$$\xi(R) = \sqrt{\frac{3}{\pi}} 2\lambda \left(1 + \frac{\lambda}{3} \right) f(\lambda), \quad \lambda = (\sigma R^2)^{2/3}, \quad (45)$$

$$f(\lambda) = \int_0^{\infty} dy \frac{e^{-\varphi(\lambda y)/y}}{\sqrt{\sinh(\lambda y)} \sinh(\sqrt{3}\lambda y)} \quad (46)$$

For $\lambda \rightarrow 0$ one has $f(\lambda) \approx \frac{1}{2\lambda} \sqrt{\frac{\pi}{3}}$, and $\xi(\lambda \rightarrow 0)$ is close to unity. The explicit behaviour of $\xi(R)$ is given in the Table.

Table : The screening factor $\xi(R)$ and $f(\lambda)$ as functions of distance R

λ	0.1	0.2	0.4	0.6	0.8	1.0	2.7	5.4
$f(\lambda)$	4.60	1.91	0.74	0.41	0.26	0.18	0.024	0.0029
R (fm)	0.084	0.14	0.23	0.32	0.39	0.46	0.98	1.645
$\xi(R)$	0.929	0.796	0.656	0.577	0.515	0.469	0.241	0.086

For $\sigma R^2 \rightarrow \infty$, one has from (34) $\bar{\mu} = \left(\frac{\sigma}{R}\right)^{1/3}$ and inserting this into (44) one obtains the asymptotics.

$$V^*(R) \approx \sqrt{\frac{2}{3\pi}} \frac{\lambda \exp(-\frac{\lambda}{2})}{R} \approx \frac{\sqrt{2}(\sigma R^2)^{2/3} \exp\left[-\frac{1}{2}(\sigma R^2)^{2/3}\right]}{\sqrt{3}\pi^{3/2}R}, \quad (\sigma R^2) \gg 1 \quad (47)$$

One can see in the Table that the screening starts at rather small values of R and at $R \sim 0.5$ fm, the coefficient $\xi(R) \sim 0.5$.

6 Conclusions

The overall static interaction between Q and \bar{Q} can be derived from Eq. (5), where the term $\langle W^{(0)}(B) \rangle$ gives the confining term $V_{conf}(R)$, while the second term on the r.h.s. of (5) provides the screened Coulomb potential $V_C^*(R)$, so that one has for $R \gg \lambda$, $\lambda \approx 0.2$ fm.

$$V_{static}(R) = \sigma R + V_C^*(R), V_C^*(R) = -\frac{C_2 \alpha_s}{R} \xi(R), \quad (48)$$

where $\xi(R)$ is given in (45),(46) and in the Table.

In (48) the interference of perturbative field a_μ and nonperturbative B_μ is taken into account. At small R , $R \lesssim \lambda$, there exists another interference effect, which was treated before in [12], and which provides linear behaviour of $V_{conf}(R)$ at very small R , while without this interference $V_{conf}(R) \sim const R^2$, for $R \lesssim \lambda$ [13].

The behaviour $\sigma R - \frac{e}{R}$ was checked on the lattice in the interval $0.1 \text{ fm} < R < 1 \text{ fm}$ with good accuracy [14] and the region $0.8 \text{ fm} < R < 1.5 \text{ fm}$ was also measured [15] in the regime where string breaking is expected. Recently a thorough analysis of the region $R \lesssim 1 \text{ fm}$ [16] has revealed that

$\xi(R)$ is approximately constant in this interval and coincides within 15% with the bosonic string Casimir energy prediction (see [16] for discussion and earlier references). Also the heavy quarkonia spectrum calculated assuming $\xi(R) \equiv 1$ is in good agreement with experiment including high excited bottomonium states [17,18] and deviation of $\xi(R)$ from unity for $R \gtrsim 0.5$ fm seems to deteriorate this agreement. Thus the calculated in this paper screening factor $\xi(R)$, Eq.(45) and Table, is in a serious conflict with lattice and phenomenological (experimental) data. A possible solution of this paradox lies in adopting the bosonic string term [16] at distances beyond 0.5 fm, so that the sum of the screened Coulomb term and the bosonic string term could imitate the original unscreened Coulomb interaction. This picture of trasmutation of the Coulomb into the string vibration term, if realistic, can be supported by the Casimir scaling study of the Coulomb-like term at distances around 1 fm. The accuracy of the previous study by Bali [14] was insufficient to draw definite conclusions about the presence of the bosonic string term in this region.

In addition one should recalculate the bosonic string term using the realistic hybrid spectrum found in [6], which will be reported elsewhere.

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Appendix 1

Derivation of the gluon Green's function in the toy model of the confining plane (chapter 3)

Eq.(19) describes Green function of free motion in the plane (x_1, x_4) and factorizable motion in oscillator potential in the directions x_2 and x_3 . For the latter one can use the standard textbook formula (see e.g. [19]) for the Green's function $G(x_a, t_a; x_b, t_b)$ corresponding to the Lagrangian

$$L = \frac{m\dot{x}^2}{2} - \frac{m\omega_0^2 x^2}{2},$$

which we write in the Euclidean metrics

$$G(x_a, t_a; x_b, t_b) = \left(\frac{m\omega_0}{2\pi \sinh \omega_0 T} \right)^{1/2} \exp \left\{ -\frac{m\omega_0}{2 \sinh \omega_0 T} [(x_a^2 + x_b^2) \cosh \omega_0 T - 2x_a x_b] \right\}$$

where $T = t_a - t_b$. Changing $m\omega_0^2 \rightarrow \frac{\omega^2}{4}$, $m = \frac{1}{2}$, and identifying $x_a = x_b = 0$, one arrives at the result written in Eq. (20).

Appendix 2

The gluon propagator in the einbein path-integral representation

One starts with FSR for the free propagator which can be written as

$$G(x, y) = \int_0^\infty ds (Dz)_{xy} \exp(-K) \quad (\text{A2.1})$$

and introduce the einbein variable–dynamical mass $\mu(t)$ as in [10], so that K can be rewritten as

$$\begin{aligned} K &= m^2 s + \frac{1}{4} \int_0^s \left(\frac{dz_\mu(\tau)}{d\tau} \right)^2 d\tau = \\ &= \int_0^T dt \left\{ \frac{m^2}{2\mu(t)} + \frac{\mu(t)}{2} + \frac{\mu(t)}{2} \left(\frac{dz_i(t)}{dt} \right)^2 \right\}. \end{aligned} \quad (\text{A2.2})$$

In $(Dz)_{xy}$, there is integration over time components of the path, namely

$$(Dz_4) \equiv \prod_n \frac{d\Delta z_4(n)}{(4\pi\varepsilon)^{1/2}} \delta \left(\sum \Delta z_4 - T \right) \quad (\text{A2.3})$$

where $T \equiv x_4 - y_4$, and using the definition of $\mu(t)$, $\mu(t) = \frac{1}{2} \frac{dt}{d\tau}$ one can rewrite the integration element in (A2.3) as follows ($t \equiv z_4$)

$$\frac{d\Delta z_4(n)}{\sqrt{4\pi\varepsilon}} = 2d\mu(n) \sqrt{\frac{\varepsilon}{\pi}} = \frac{d\mu(n) \sqrt{\Delta t}}{\sqrt{2\pi\mu(n)}}, \quad \sqrt{\varepsilon} = \sqrt{\frac{\Delta t}{2\mu(n)}}. \quad (\text{A2.4})$$

Moreover the δ -function acquires the form

$$\delta\left(\sum \Delta z_4 - T\right) = \delta(s2\bar{\mu} - T) \quad (\text{A2.5})$$

where we have defined

$$\bar{\mu} = \frac{1}{s} \int_0^s 2\mu(\tau) d\tau. \quad (\text{A2.6})$$

As a result one can integrate in (A2.1) over ds using δ -function (A2.5), and rewrite $ds(Dz)_{xy}$ as $ds(D^4z)_{xy} = (D^3z)_{\mathbf{x}\mathbf{y}} D\mu$. One can write the Green's function as follows

$$G(x, y) = \int \prod \frac{d^3 \Delta z_i(n)}{l^3} e^{-K} \frac{d\mu(n)}{l_\mu(n)} \frac{d^3 p}{(2\pi)^3} e^{i\mathbf{p}(\mathbf{x}-\mathbf{y}-\sum \Delta \mathbf{z}(n))} \quad (\text{A2.7})$$

where K is given in (A2.2), and l, l_μ are

$$l(n) = \left(\frac{2\pi\Delta t}{\mu(n)}\right)^{1/2}, \quad l_\mu(n) = \left(\frac{2\pi\mu(n)}{\Delta t}\right)^{1/2}, \quad N\Delta t = x_4 - y_4 \equiv T.$$

The integration over $d^3 \Delta z_i(n)$ yields

$$G(x, y) = \int \frac{d^3 p}{(2\pi)^3} e^{i\mathbf{p}(\mathbf{x}-\mathbf{y}) - \frac{1}{2} \int_0^T dt \mu(t) \left(1 + \frac{\mathbf{p}^2 + m^2}{\mu^2(t)}\right)} \frac{1}{2\bar{\mu}} (D\mu). \quad (\text{A2.8})$$

Taking into account the integral

$$\int_0^\infty \frac{d\mu(n)}{\sqrt{\mu(n)}} e^{-\frac{\Delta t}{2} \left(\mu(n) + \frac{\mathbf{p}^2 + m^2}{\mu(n)}\right)} = \sqrt{\frac{2\pi}{\Delta t}} e^{-\Delta t \sqrt{\mathbf{p}^2 + m^2}} \quad (\text{A2.9})$$

one has finally

$$G(x, y) = \int \frac{d^3 p}{(2\pi)^3} \frac{e^{i\mathbf{p}(\mathbf{x}-\mathbf{y}) - \int_0^T dt \sqrt{\mathbf{p}^2 + m^2}}}{2\sqrt{\mathbf{p}^2 + m^2}} \quad (\text{A2.10})$$

where we have used relation following from the stationary point in the integral (A2.9)

$$\bar{\mu} = \int_0^s \mu(\tau) d\tau = \sqrt{\mathbf{p}^2 + m^2}. \quad (\text{A2.11})$$

This can be compared with the integral

$$G(\mathbf{r}, T) = \int \frac{d^4 p}{(2\pi)^4} \frac{e^{i\mathbf{p}\mathbf{r} + ip_4 T}}{p_4^2 + \mathbf{p}^2 + m^2}, \quad \mathbf{r} = \mathbf{x} - \mathbf{y}, \quad (\text{A2.12})$$

which reduces to (A2.10) after integrating over dp_4 for $T > 0$. Eq. (A2.12) is the standard form of the free propagator. Now in the case of interacting gluon as in section 4 one can still use (A2.5), (A2.6) with the result, that $\bar{\mu} = \mu_0$, with μ_0 defined in (34) since μ_0 does not depend on time.

References

- [1] B.S.De Witt, Phys. Rev. **162**, 1195, 1239 (1967)
J.Honerkamp, Nucl. Phys. B **48**, 269 (1972);
G.'t Hooft Nucl. Phys. B **62**, 444 (1973), *Lectures at Karpacz, in : Acta Univ. Wratislaviensis* **368**, 345 (1976);
L.F.Abbot, Nucl. Phys. B **185**, 189 (1981).
- [2] Yu.A.Simonov, Phys. At. Nucl. **58**, 107 (1995); Yad. Fiz. **58**, 113 (1995).
- [3] Yu.A.Simonov, in: *Lecture Notes in Physics, v.479, p.144, Springer, 1996*.
- [4] A.B.Kaidalov and Yu.A.Simonov, Phys. Lett. B **477** 163 (2000);
hep-ph/9912434;
A.B.Kaidalov and Yu.A.Simonov, Phys. Atom. Nucl. **63**, 1428 (2000);
Yad. Fiz. **63** 1428 (2000); hep-ph/9911291
- [5] Yu.A.Simonov, Nucl. Phys. B (proc. Suppl) **23**, 283 (1991);
Yu.S.Kalashnikova, Yu.B.Yufryakov, Phys. Lett.B **359**, 175 (1995).
- [6] Yu.S.Kalashnikova, D.S.Kuzmenko, Phys. Atom. Nucl. **64**, 1716 (2001);
ibid. **66**, 955 (2003), hep-ph/0203128; hep-ph/03022070.
- [7] Yu.A.Simonov, Nucl. Phys. B **592**, 350 (2001); hep-ph/0003114.
- [8] A.Di Giacomo and H.Panagopoulos, Phys. Lett. B **285**, 133 (1992);
A.Di Giacomo, E.Meggiolaro and H.Panagopoulos, Nucl. Phys. B **483**,
371 (1997);
M.D'Elia, A.Di Giacomo and E.Meggiolaro Phys.Lett. B **408**, 315
(1997).
- [9] K.J.Juge, J.Kuti and C.Morningstar, Nucl. Phys. Proc. Suppl. **73**, 360
(1999); hep-lat/9809015; Phys. Rev. Lett. **90**, 161601 (2003).
- [10] A.Yu.Dubin, A.B.Kaidalov, and Yu.A.Simonov, Phys. Lett. B **323**, 41
(1994);
A.Yu.Dubin, A.B.Kaidalov, Yu.A.Simonov, Phys. At. Nucl. **56**, 1745
(1993); hep-ph/9311344.

- [11] Yu.A.Simonov and J.A.Tjon, Ann Phys. **228**, 1 (1993);
Yu.A.Simonov, J.A. Tjon, Annals Phys. **300**, 54 (2002);
hep-ph/0205165.
- [12] Yu.A.Simonov, Phys. Rept. **320**, 265 (1999); JEPT Lett **69**, 505 (1999).
- [13] H.G.Dosch, Phys. Lett. B **190**, 177 (1987);
H.G.Dosch and Yu.A.Simonov, Phys. Lett. B **205**, 339 (1988);
Yu.A.Simonov, Nucl. Phys. B **307**, 512 (1988).
- [14] G.Bali, Nucl. Phys. Proc. Suppl. **83**, 422 (2000), Phys. Rev. D**62**, 114503 (2000);
A.Hasenfratz, R.Hoffmann and F.Knechtli, Nucl. Phys. Proc. Suppl. **106**, 418 (2002).
- [15] B.Bolder, T.Struckmann, G.S.Bali, N.Eicker et al., Phys. Rev. D**63**, 074504 (2001).
- [16] M.Lüscher and P.Weisz, JHEP 0207, 049 (2002).
- [17] E.Eichten et al. Phys. Rev. D **21**, 203 (1980);
E.J.Eichten, K.Lane and C.Quigg, Phys. Rev. Lett. **89**, 162002 (2002).
- [18] A.M.Badalian, B.L.G.Bakker, Phys. Rev. D **67**, 071901 (2003);
A.M.Badalian, B.L.G.Bakker and A.I.Veselov, Phys. Rev. D (in press),
hep-ph/0311010.
- [19] R.P.Feynmann, A.R.Hibbs, *Quantum Mechanics and Path Integrals*
Mc.Graw-Hill, N.Y., 1965, problem 3.8.